Pergunta 2.

X – variável aleatória que representa o desvio do comprimento especificado de uma peça (em cm)

a.

$$E[X] = \int_{-\infty}^{+\infty} x\, f(x)\ dx = \int_{-\infty}^{-1} 0\, dx + \int_{-1}^{0} x\,(1+x)dx + \int_{0}^{1} x\,(1-x)dx + \int_{1}^{+\infty} 0\, dx$$

$$= \left[\frac{x^2}{2}+\frac{x^3}{3}\right]_{-1}^{0} + \left[\frac{x^2}{2}-\frac{x^3}{3}\right]_{0}^{1} = 0 - \left(\frac{1}{2}-\frac{1}{3}\right) + \left(\frac{1}{2}-\frac{1}{3}\right) - 0 = 0\ \Leftrightarrow E[X] = 0$$

Tem-se $\quad Var[X] = E[X^2] - (E[X])^2$

$$E[X^2] = \int_{-\infty}^{+\infty} x^2\, f(x)dx = \int_{-\infty}^{1} 0\, dx + \int_{-1}^{0} x^2\,(1+x)dx + \int_{0}^{1} x^2\,(1-x)dx + \int_{1}^{+\infty} 0\, dx =$$

$$= \left[\frac{x^3}{3}+\frac{x^4}{4}\right]_{-1}^{0} + \left[\frac{x^3}{3}-\frac{x^4}{4}\right]_{0}^{1} = 0 - \left(-\frac{1}{3}+\frac{1}{4}\right) + \left(\frac{1}{3}-\frac{1}{4}\right) - 0 = \frac{1}{6}$$

Logo

$$Var[X] = \frac{1}{6}$$

b.

$$F(x) = \int_{-\infty}^{\mathrm{x}} f(t)\mathrm{dt}$$

$$x < -1, \qquad\qquad F(x) = 0$$

$$-1 \le x < 0, \quad F(x) = \int_{-\infty}^{-1} 0\,\mathrm{dt} + \int_{-1}^{\mathrm{x}} (1+\mathrm{t})\,\mathrm{dt} = \left[t+\frac{t^2}{2}\right]_{-1}^{x} = x + \frac{x^2}{2} + \frac{1}{2}$$

$$-0 \le x \le 1, \quad F(x) = \int_{-\infty}^{-1} 0\,\mathrm{dt} + \int_{-1}^{0} (1+\mathrm{t})\,\mathrm{dt} + \int_{0}^{x} (1-t)\,dt = \frac{1}{2} + \left[t-\frac{t^2}{2}\right]_{0}^{x}$$

$$= \frac{1}{2} + x - \frac{x^2}{2}$$

$$x > 1, \qquad F(x) = F(1) + \int_{1}^{x} 0\ dt = 1$$

Logo

$$F(x) = \begin{cases} 0 & , \quad x < -1 \\ x + \frac{x^2}{2} + \frac{1}{2} & , \quad -1 \leq x < 0 \\ x - \frac{x^2}{2} + \frac{1}{2} & , \quad 0 \leq x \leq 1 \\ 1 & , \quad x > 1 \end{cases}$$

c.

Terceiro quartil é, por definição, o **quantil de probabilidade 0.75,** $X_{0,75}$, isto é, F($X_{0,75}) = 0.75$.

Como $F(0.5) = 0.5 - \frac{0.5^2}{2} + \frac{1}{2} = 0.875$ e F é uma função crescente em $\mathbb{R} \Rightarrow X_{0,75}$ <0,5

d.

$$Y = 2 - 2X^2$$

$$P[Y < 1] = P[2 - 2X^2 < 1] = P[-2X^2 < 1] = P\left[X^2 > \frac{1}{2}\right] = P\left[X > \sqrt{\frac{1}{2}} \quad \vee \quad X < -\sqrt{\frac{1}{2}}\right]$$

$$= P\left[X > \sqrt{\frac{1}{2}}\right] + P\left[X < -\sqrt{\frac{1}{2}}\right] = 2\left(P\left[X < -\sqrt{\frac{1}{2}}\right]\right) = 2\left(-\sqrt{\frac{1}{2}} + \frac{\frac{1}{2}}{2} + \frac{1}{2}\right)$$

$$\cong 0.085786$$